CM01 UNIFORM CIRCULAR MOTION

SPH4U

EQUATIONS

- Centripetal Acceleration

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

- Uniform Circular Motion

$$v = \frac{2\pi r}{T}$$

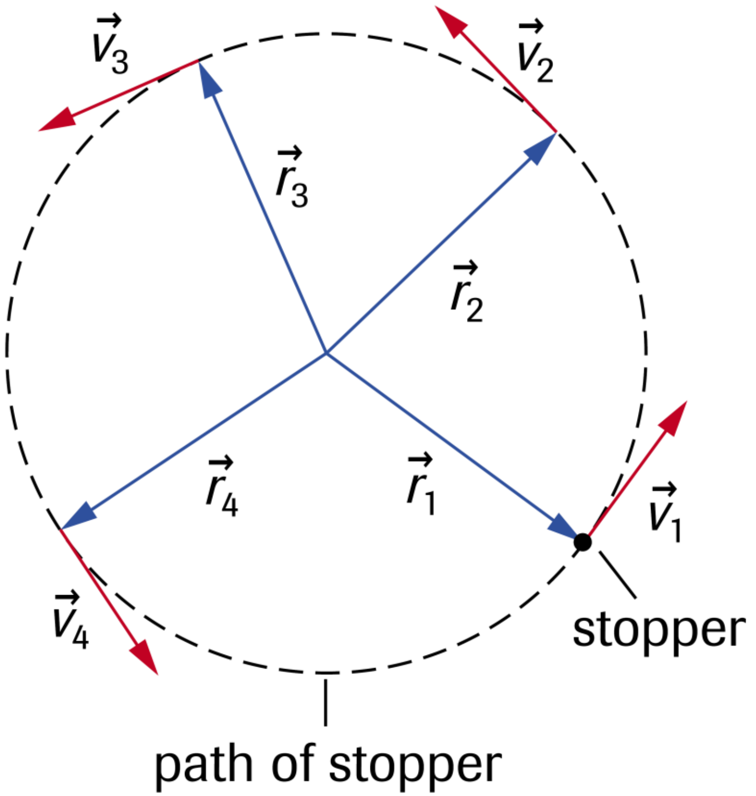
- Frequency

$$f = \frac{1}{T}$$

UNIFORM CIRCULAR MOTION

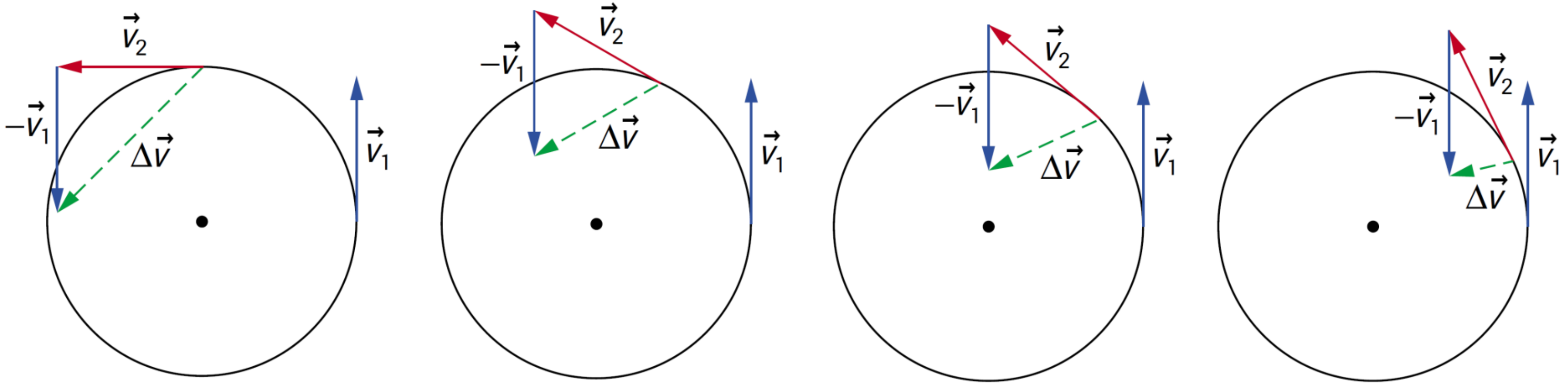
- **Uniform Circular Motion:** motion that occurs when an object has constant speed and constant radius

- **Centripetal Acceleration:** instantaneous acceleration directed toward the centre of the circle



DIRECTION OF CENTRIPETAL ACCELERATION

- Recall: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$
  - as the time interval decreases, $\Delta \vec{v}$ points closer to the centre of the circle
  - since $\vec{a}$ is in the same direction as $\Delta \vec{v}$, we can conclude that **the direction of centripetal acceleration is towards the centre.**



MAGNITUDE OF CENTRIPETAL ACCELERATION

- *(See pg 123-124 for proof)*
- Magnitude of centripetal acceleration

$$a_c = \frac{v^2}{r}$$

  - a_c – centripetal acceleration
  - v – uniform speed
  - r – radius of circle (or arc)

EXAMPLE 1

A child on a merry-go-round is 4.4 m from the centre of the ride, travelling at a constant speed of 1.8 m/s. Determine the magnitude of the child's centripetal acceleration.

EXAMPLE 1 – SOLUTIONS

$v = 1.8$ m/s

$r = 4.4$ m

$a_c = ?$

$$a_c = \frac{v^2}{r}$$

$$= \frac{(1.8 \text{ m/s})^2}{4.4 \text{ m}}$$

$$a_c = 0.74 \text{ m/s}^2$$

The child's centripetal acceleration has a magnitude of 0.74 m/s^2.

SPEED OF UNIFORM CIRCULAR MOTION

- Often speed is not known
- More often, we know radius (r) and period (T)
  - The period is the time to complete one full revolution
- Speed is constant, and by using the circumference of the circle we get

$$v = \frac{2\pi r}{T}$$

which results in

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

FREQUENCY VS PERIOD

- For higher rates of revolution, we use frequency instead of period
  - **Frequency** (f) [Hz $=$ s^{-1}]: the number of revolutions per second

$$f = \frac{1}{T}$$

- In terms of frequency, we get

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

EXAMPLE 2

Find the magnitude and direction of the centripetal acceleration of a piece of lettuce on the inside of a rotating salad spinner. The spinner has a diameter of 19.4 cm and is rotating at 780 rpm (revolutions per minute). The rotation is clockwise as viewed from above. At the instant of inspection, the lettuce is moving eastward.

EXAMPLE 2 – SOLUTIONS

$f = (780 \text{ rev/min})(1 \text{ min/60 s}) = 13 \text{ Hz} = 13 \text{ s}^{-1}$

$r = \frac{19.4 \text{ cm}}{2} = 9.7 \text{ cm} = 9.7 \times 10^{-2} \text{ m}$

$\vec{a}_c = ?$

$$a_c = 4\pi^2 r f^2$$
$$= 4\pi^2 (9.7 \times 10^{-2} \text{ m})(13 \text{ s}^{-1})^2$$
$$a_c = 6.5 \times 10^2 \text{ m/s}^2$$

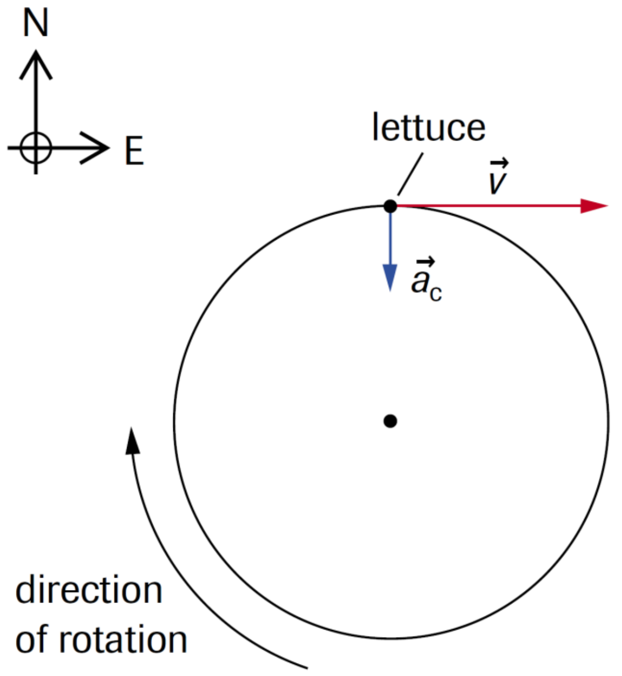


Figure 5

Figure 5 shows that since the lettuce is moving eastward, the direction of its centripetal acceleration must be southward (i.e., toward the centre of the circle). The centripetal acceleration is thus 6.5×10^2 m/s^2 [S].

SUMMARY:
UNIFORM CIRCULAR MOTION

- Uniform circular motion is motion at a constant speed in a circle or part of a circle with a constant radius.
- Centripetal acceleration is the acceleration toward the centre of the circular path of an object travelling in a circle or part of a circle.
- Vector subtractions of position and velocity vectors can be used to derive the equations for centripetal acceleration.

PRACTICE

Readings

- Section 3.1 (pg 122)

Questions

- pg 127 #2-4,6,7